A Study of Supersonic Turbulence in Stagnating Plasma

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How Z-pinch works



The $\vec{J} \times \vec{B}$ Lorentz force makes the plasma implode.



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Implosion

Stagnation

Most of the X-ray emission takes place at the stagnation phase.

"Z machine" (Sandia Labs, US)



Z-pinch in Weizmann Inst. (Israel)



Pinches as laboratory astrophysics

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"A simple arrangement of electric and magnetic fields causes plasma to form shapes reminiscent of the jets generated near supermassive black holes."

Pinching is a naturally occuring phenomenon, but importance of z-pinches as an astrophysical laboratory goes well beyond that.

[You et al., 2005]

Pinches as laboratory astrophysics: im/explosions

[Foord et al., 1994]



FIG. 2. A typical OIII line profile ($A_0 = 3047.1$ Å measured radially at z = 11 mm and t' = -150 ns. Redshifted and blueshifted components from each side of the annulus are observed. Doppler shifts correspond to a radial velocity ≈ 3 cm/ μ s.

Pinches as laboratory astrophysics: im/explosions

Z-pinch ($r \sim 1 \text{ cm}$)



[Foord et al., 1994]



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Pinches as laboratory astrophysics: instabilities



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Pinches as laboratory astrophysics: instabilities





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Energy conversion in imploding plasmas

Imploding plasmas are promising candidates for fusion (NIF, MagLIF) and unique sources of intense x-ray radiation (z-pinches).







Thus, one needs to measure ion T_i . It is also important to know the hydro energy.

Principal difficulty:

The Doppler broadening (also, neutron spectrum) gives information only on the total ion velocity distribution $\rightarrow T_i^{\text{eff}} \ge T_i$.

Importance of distinguishing between T_i^{eff} and T_i

Assuming $T_i = T_i^{\text{eff}}$ may result in crucially misinterpreted data.

At WIS, we have succeeded developing advanced diagnostics capable of telling T_i^{eff} and T_i apart.

Diagnostics setup

Three time-resolved data sources: spectra of Ne Ly- α dielectronic satellites, gated x-ray pinhole imaging, and an absolutely calibrated photo-conductive detector (PCD) sensitive to $\hbar \omega \gtrsim 700 \text{ eV}$.



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 - Detailed energy-balance analysis [Kroupp et al., 2011, Maron et al., 2013]; or
 - Effect of Γ_{ii} on Stark lineshapes of high-*n* transitions [Alumot et al., 2017] (in preparation).

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This modeling described all the data very well, within 1 - 2 std. dev.

Determination of n_e and T_i^{eff}



Widths and intensities of the Ne Ly- α satellites allow for determining T_i^{eff} and n_e , respectively. In this example, $T_i^{\text{eff}} = 1200 \text{ eV}$ and $n_e = (5 \pm 1) \times 10^{20} \text{ cm}^{-3}$.

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 $I_{2p^2(^3P) \rightarrow 1s2p(^3P)}/I_{2s2p(^3P) \rightarrow 1s2s(^3S)}$ intensity ratio *R* is sensitive to n_e but practically independent of T_e [Seely, 1979, Kroupp et al., 2007].

Determination of T_i during z-pinch stagnation

Two methods have been used:



1315

[Alumot et al., 2017] (in preparation)

1320

1325

0.2

0.0

1310



Both methods give the same, consistent results.

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Were supersonic turbulence present, it would imply substantial nonuniformity in quantities such as the density. However, the previous analysis assumed a uniform plasma. The data need to be re-analyzed assuming a physically sound model of turbulence. [Kroupp et al., 2017]

Instead of $n_e = n_e^0 = \text{const}$, there is now a probability distribution function (PDF) $P(n_e)$ – actually, $P(t, z; n_e)$.

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Let us switch to dimensionless quantity

$$\xi \equiv n_e/n_e^0; \int P(\xi) \,\mathrm{d}\xi = 1.$$

The average density is

$$\langle n_e \rangle = n_e^0 \int \xi P(\xi) \,\mathrm{d}\xi.$$

(note that $\langle n_e \rangle \neq n_e^0$).

Assuming the collisional-radiative equilibrium is established much faster than the hydromotion, the intensity of a spectral line (or continuum radiation) is [Stamm et al., 2017]

$$\langle I \rangle = \int \alpha(\vec{r}) d^3r = \pi r_{\rm pl}^2 \ell \int \alpha(\xi) P(\xi) \, \mathrm{d}\xi,$$

where $\alpha \propto \xi^2$ is the local plasma emissivity, and $r_{\rm pl}$ and ℓ is the radius and length of the plasma segment, respectively.

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$$\left(1 - \frac{\delta I_{\rm PCD}}{I_{\rm PCD}}\right) \left(\frac{r_{\rm pl}^0}{r_{\rm max}}\right)^2 \le \int \xi^2 P(\xi) \, \mathrm{d}\xi \le \left(1 + \frac{\delta I_{\rm PCD}}{I_{\rm PCD}}\right) \left(\frac{r_{\rm pl}^0}{r_{\rm min}}\right)^2 \, .$$

Density determination:



Use linearization $R \approx R^0 + a_R(n_e/n_e^0 - 1)$, so $\langle R \rangle = R^0 + a_R \frac{\int (\xi - 1)\xi^2 P(\xi) d\xi}{\int \xi^2 P(\xi) d\xi}$.

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The measured quantity R_{expt} is known – and should remain – within its error bars, i.e., $\langle R \rangle = R_{\text{expt}} = R^0 \pm \delta R$. Therefore,

$$1 - \frac{\delta R}{a_R} \le \frac{\int \xi^3 P(\xi) \,\mathrm{d}\xi}{\int \xi^2 P(\xi) \,\mathrm{d}\xi} \le 1 + \frac{\delta R}{a_R} \,.$$

Constraints on $P(\xi)$

To summarize:

$$\left(1 - \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}\right) \left(\frac{r_{\text{pl}}^0}{r_{\text{max}}}\right)^2 \le \int \xi^2 P(\xi) \, \mathrm{d}\xi \le \left(1 + \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}\right) \left(\frac{r_{\text{pl}}^0}{r_{\text{min}}}\right)^2 \,.$$
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Once $P(\xi)$ is determined, the model plasma radius is corrected:

$$r_{\rm pl} = r_{\rm pl}^0 / \sqrt{\langle \xi^2 \rangle}$$
.

Constraints on $P_V(\xi_V)$ and β

The last step is to use the volumetric density distribution:

$$\int P_V(\xi_V) \,\mathrm{d}\xi_V = 1, \int \xi_V P_V(\xi_V) \,\mathrm{d}\xi_V = 1.$$

Introduce $\beta \equiv \xi/\xi_V = \langle n_e \rangle/n_e^0$; so $\langle \xi^k \rangle = \beta^k \langle \xi_V^k \rangle$.

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Finally:

$$\begin{split} \sqrt{\frac{1 - \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}}{\langle \xi_V^2 \rangle}} \frac{r_{\text{pl}}^0}{r_{\text{max}}} \le \beta \le \sqrt{\frac{1 + \frac{\delta I_{\text{PCD}}}{I_{\text{PCD}}}}{\langle \xi_V^2 \rangle}} \frac{r_{\text{pl}}^0}{r_{\text{min}}} \\ \left(1 - \frac{\delta R}{a_R}\right) \frac{\langle \xi_V^2 \rangle}{\langle \xi_V^3 \rangle} \le \beta \le \left(1 + \frac{\delta R}{a_R}\right) \frac{\langle \xi_V^2 \rangle}{\langle \xi_V^3 \rangle} \\ r_{\text{pl}} = r_{\text{pl}}^0 / (\beta \sqrt{\langle \xi_V^2 \rangle}) \end{split}$$

Turbulence density PDF

Volumetric PDF of [Hopkins, 2013]:

$$P_{V}(\xi_{V}) \,\mathrm{d}\xi_{V} = \frac{\mathrm{I}_{1}\left(2\sqrt{\lambda\omega(\xi_{V})}\right)}{\exp[\lambda + \omega(\xi_{V})]} \,\sqrt{\frac{\lambda}{\theta^{2}\omega(\xi_{V})}} \frac{\mathrm{d}\xi_{V}}{\xi_{V}}$$



Here,
$$\begin{split} \lambda &\equiv (1+\theta)^{3/2} \ln \left(1+M_c^2\right)/2\theta^2 \\ \omega(\xi_V) &\equiv \lambda/(1+\theta) - \ln(\xi_V)/\theta \\ \theta &\approx 0.05M_c \end{split}$$

Compressive Mach number $M_c = bM, b \approx 0.4$

 I_1 – modified Bessel function of the first kind

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Results: model mean density and plasma radius



The mean plasma density is inferred to be lower by a factor ~ 2 .

Results: model mean density and plasma radius

The corrected plasma model radius fits the data better.



Results: model mean density and plasma radius



The other plasma parameters (T_e , T_i , and T_i^{eff}) remain unaffected.

Results in a wider scientific context

In addition to better understanding of z-pinch stagnation plasmas, a crucial question arises:

Is the supersonic turbulent hydromotion generated and carried along during the implosion phase?

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If yes (and we have preliminary results confirming it), then z-pinches represent a test bed for:

- a recently proposed novel fast ignition scheme [Davidovits and Fisch, 2016] for inertial confinement;
- astrophysical phenomena, such as molecular cloud dynamics, star formation efficiency, the core mass/stellar initial mass function, and more.

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- This turbulent-plasma model is not only consistent with the observations, it improves the agreement with them (*r*_{pl}).
- Beyond aiding our understanding of z-pinches, we hope this study can provide fertile ground for dealing with related problems of astrophysical interest.

Thank you!

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Extra material

The experimental data [Kroupp et al., 2011] relevant for the analysis presented; the plasma parameters assumed for (r_{pl}^0, n_e^0, T_e) and inferred from (T_i, M, Re) the *uniform*-plasma modeling; the calculated isothermal turbulence parameters, volumetric density factor β and respectively corrected plasma electron density and radius. Units are as follows: all radii are in mm, all temperatures are in eV, and densities are in 10^{20} cm^{-3} .

Experimental data						Uniform plasma						Isothermal turbulence						
t (ns)	δR	$I_{\rm PCD}$ (GW)	r_{\min}	$r_{\rm max}$	T_i^{eff}	r ⁰ _{pl}	n_e^0	T_e	T_i	М	Re	θ	$\sigma^2_{s,V}$	$\langle \xi_V^2 \rangle$	$\langle \xi_V^3 \rangle$	β	$n_e^{ m turb}$	r ^{turb}
-3.4	0.15	0.35 ± 0.3	0.19	0.41	3000	0.23	6.0	120	250	2.4	8.1×10^{4}	0.048	0.70	1.84	5.77	0.32	1.9	0.53
-2.0	0.15	2.0 ± 1.0	0.25	0.47	2100	0.29	6.0	175	230	1.7	6.9×10^4	0.034	0.40	1.44	2.86	0.54	3.2	0.45
-1.2	0.15	3.8 ± 1.1	0.36	0.52	1800	0.31	6.0	190	210	1.6	7.7×10^4	0.032	0.36	1.39	2.60	0.60	3.6	0.44
0.0	0.15	6.5 ± 0.7	0.46	0.68	1300	0.35	6.0	185	200	1.3	8.9×10^4	0.026	0.25	1.26	1.96	0.57	3.4	0.55
2.0	0.15	3.6 ± 1.0	0.36	0.53	900	0.24	6.0	155	180	1.2	$7.4 imes 10^4$	0.024	0.21	1.22	1.80	0.53	3.2	0.41
3.3	0.15	2.3 ± 0.9	0.21	0.43	720	0.20	6.0	140	180	1.0	5.1×10^4	0.020	0.15	1.16	1.53	0.62	3.7	0.30

Results: turbulent density PDF's



Is turbulence in this stagnating plasma isothermal?

Compare v_{flow} to a thermal conduction velocity (following [Zeldovich and Raizer, 1967]),

$$v_{\rm cond} = \frac{L_h}{\tau_{\rm cond}} \approx 4 \times 10^{21} \frac{\zeta(\langle Z_i \rangle)}{(\langle Z_i \rangle + 1)\lambda_{ei}} \frac{T^{5/2}}{n_e L_h},$$

where L_h is a length scale $(L_h \sim r_{\rm pl})$, λ_{ei} is the Coulomb logarithm, and $\zeta(8.5) \approx 2.7$; T is in units of eV, n_e in cm⁻³, and L_h in cm.

When $v_{\rm cond}/v_{\rm flow} \gg 1$, isothermality is expected.

 $v_{\text{cond}}/v_{\text{flow}} \sim 2$ for $L_h \sim r_{\text{pl}}$ at t = -3.4 ns, and ~ 6 for later times.

An accurate determination of the degree of isothermality would require detailed simulations. The inferred T_e will also need to be reconsidered, since the emissivity depends on T_e strongly.

Interestingly, in Sandia Z experiments, where T_e and $\langle Z_i \rangle$ reach higher values, $v_{\text{cond}}/v_{\text{flow}} \gg 1$, thus the assumption of turbulence isothermality should be fully justified.