Modeling of hydrogen Balmer lines for the diagnostic of magnetic white dwarf atmospheres

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1) Presentation of white dwarfs

2) Stark broadening calculations in WD atmosphere conditions

3) Zeeman effect in magnetized white dwarfs
White dwarfs: an overview

- WD are the end of the majority of stars (95 – 97%) with $M < 10M_\odot$.

- About 10% of WD have strong magnetic field.

- They have a stratified structure:
  - C, O core (99% M)
  - thin mantle of He (1% M)
  - envelope of H (< 0.01% M)

- They are classified by their dominant element in the atmosphere:
  - DA: strong hydrogen lines
  - DB: strong He I lines
  - DO: strong He II lines
  etc.


Source: S. L. Shapiro and S. A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars
Example of white dwarf spectrum

Sloan Digital Sky Survey
http://www.sdss.org

SDSS J165538.93+253345.99

Data from Belgrade Observatory (J. Kovačević-Dojčinović, M. S. Dimitrijević, L. Č. Popović)
Absorption lines in WD atmospheres

The outgoing radiation spectrum is obtained by solving the radiative transfer equation

\[
\frac{dI_\nu}{ds} = \eta_\nu - \kappa_\nu I_\nu \quad (+ \text{scattering})
\]
Modeling the extinction coefficient

\[ K_v = K_v^{ff} + K_v^{bf} + K_v^{bb} \]

Free-free transitions: inverse bremsstrahlung, Rayleigh scattering, Thomson scattering

Bound-free transitions: photoionization

Bound-bound transitions: photoexcitation (atomic lines)
The bound-bound extinction coefficient

The depth of the absorption lines is determined by the bound-bound extinction coefficient

\[ \kappa_{vb} = \sum_{lu} \frac{h \nu}{4 \pi} B_{lu} N_l \phi_{lu, \nu} \]

atomic population

line shape
Line broadening mechanisms

Wikipedia:
“A spectral line extends over a range of frequencies, not a single frequency”

Some causes of line broadening:
- radiative decay (natural broadening)
- Doppler effect (thermal motion of atoms)
- collisions, Stark effect –d.E
Stark broadening in stellar atmosphere conditions

Doppler broadening
Stark broadening

Normalized line shape

$H_\alpha$
$N_e = 10^{17} \text{ cm}^{-3}$
$T = 1 \text{ eV}$
Stark broadening modeling

When emitting or absorbing a photon, an atom feels the presence of the charged particles located at vicinity

A Stark broadened line is proportional to the Fourier transform of the atomic dipole autocorrelation function

\[ I(\omega) \propto \frac{1}{\pi} \text{Re} \int_{0}^{\infty} \left\langle \vec{d}(0) \cdot \vec{d}(t) \right\rangle e^{i\omega t} dt \]
Stark broadening modeling

H Lyman α, simulation
N = 10^{17} \text{ cm}^{-3}, \text{ T} = 1 \text{ eV}

Decrease time \sim 1/\Delta\omega_{1/2} \quad \text{“time of interest”}
Calculation methods

Many models, formulas and codes have been developed:
- quasistatic approximation (-d.E = cst)
- kinetic theory
- collision operators
- stochastic processes (MMM, FFM)
- fully numerical simulations

They are complementary to each other

Their validity can be assessed through comparisons to experimental spectra,
and by cross-checking between codes (e.g. SLSP code comparison workshop, Vrdnik, last week)
Lyman $\alpha$
$N_{e,i} = 10^{19} \text{ cm}^{-3}$
$T_{e,i} = 5 \text{ eV}$

Calculation methods

SLSP5 (last week)
Fitting an observed spectrum

A simplified atmosphere model: homogeneous medium

Beer-Lambert formula

\[ \phi_\nu \propto -\ln\left( \frac{I_\nu}{I_0} \right) \]

- \ln(I/I_0)

-10
0
10
20
30
40
50

-0.05
0.00
0.05

\Delta \omega (\text{eV})

Observed spectrum
Adjustment

H\alpha

\[ N_e = 6 \times 10^{17} \text{ cm}^{-3} \]

T = 1 eV
Influence of an external magnetic field on spectral lines

Zeeman effect: the energy levels and corresponding spectral lines are split

\[ \mu B \]
Zeeman effect in magnetic white dwarf spectra

Data from Belgrade Observatory (J. Kovačević-Dojčinović, M. S. Dimitrijević, L. Č. Popović)

The separation between the components corresponds to $B = 360 \, \text{T}$
At very strong magnetic fields, the Zeeman triplet structure is no longer symmetrical

\[ \frac{1}{2m_e} \left( \vec{p} + e\vec{A} \right)^2 = \frac{p^2}{2m_e} - \vec{\mu} \cdot \vec{B} + \frac{e^2 \vec{A}^2}{2m_e} \]

**Quadratic Zeeman effect**

linear Zeeman effect

quadratic Zeeman effect
Quadratic Zeeman effect

\[ H_\alpha \]
\[ N_e = 10^{17} \text{ cm}^{-3} \]
\[ T_e = T_i = 1 \text{ eV} \]
\[ B = 2000 \text{ T} \]
\[ \theta = 90^\circ \]
Quadratic Zeeman effect

\[ H_B \]
\[ N_e = 10^{17} \text{ cm}^{-3} \]
\[ T_e = T_i = 1 \text{ eV} \]
\[ B = 2000 \text{ T} \]
\[ \theta = 90^\circ \]
Quadratic Zeeman effect

![Graph showing line shapes and energy levels (eV)]
Observation on magnetic white dwarf spectra

SDSS database

Hα Zeeman components
Summary

White dwarf spectra contain information on the plasma parameters.

Accurate models are required for line broadening: Stark effect, Zeeman effect.

Ongoing work: quadratic Zeeman effect.