Collisional contribution to the Spectral line shape in magnetized plasmas

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* The collisional contribution to the spectral line shape is modified by the presence of the magnetic field, in the case of electronics perturbers, the resolution of the stochastic equation is based on the theory of impact of the interaction [M. Baranger].

** In the standard model, the emitter is subjected to a succession of independent collisions carried out by the electrons,

*** The effect of electrons perturbers is presented by a phenomenological operator of electronic collisions [H. Huddlestone et al], which can be calculated by the relaxation method [Griem et al],

**** This collision operator $\Phi(\vec{v}, \vec{B})$ must take into account the influence of the magnetic field on the collision: the trajectory of the perturbers is modified in the presence of the magnetic field, as well as the velocity distribution function.
The collisional contribution to the profile is also modified by the magnetic field.

When the perturbers are electrons, the solution of the stochastic equation is usually achieved by using the impact theory of the interaction [M. Baranger].

In the standard model, the emitter supplied to successive independent collisions with electrons.

The collective picture of the interactions with the electrons is generally described by a screened potential with Debye length.

The solution of the stochastic equation is given in the impact regime: corresponding to a long time of interest greater than the mean collision time.

In the presence of magnetic field:

The collision operator must take in consideration the influence of $B$ on the collision:

- in fact the perturbers trajectory is modified, and the velocities distribution too.

- The trajectory of electron are helicoidal parallel to $B$, and Larmor radius
Define **Larmor mean radius** $\rho_L$ corresponding to the most probable movement of the perturbers with thermal velocity $v$

$$\rho_L = \frac{m_e}{e|\mathbf{B}|}v$$

$$v = \sqrt{\frac{2kT}{m_e}}$$

In the impact theory, the **Debye length** $\rho_D$ constitutes the upper limit on the impact parameters of the cross sections of the collisions

$$\rho_D = \sqrt{\frac{kT}{4\pi N_e q_e^2}}$$

**If $\rho_D$ is greater than $\rho_L$:**

The influence of the trajectory curvature is not negligible in the presence of the magnetic field
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Debye Length:

\[ \rho_D = \sqrt{\frac{kT}{4\pi N_e e^2}} \]

Larmor radius:

\[ \rho_L = \frac{m_e c}{e |B|} \]

The ratio \( \rho_D / \rho_L \) is independent of the temperature.

In this figure, the lines show this ratio in the diagram \((N_e, B)\). This diagram shows the non consideration of the trajectory effect is valid only when the radius of Larmor \( \rho_L \) is greater than the length of Debye \( \rho_D \)(laboratory plasmas), for the rest of the cases, especially at low densities \( N_e < 5 \times 10^{14} \) cm\(^{-3} \) and for very strong fields \( B > 10^5 \) G (astrophysical plasmas) the influence of the curvature of the trajectories is not negligible and then we have to take it into account.
The effects of electron collisions on the profil are treated in the impact approximation:

- a perturbative expansion (Dyson series) [H. Huddlestone et al] of the system emitter-perturbers
- a multipolar development of the interaction allows to write

\[
\Phi = -\pi N_e \int v f(v) dv \left[ \rho_{\text{min}}^2 + \frac{4}{3} \left( \frac{\hbar}{m} \right)^2 \frac{\mathbf{r} \cdot \mathbf{r}}{v^2} \ln \frac{\rho_{\text{max}}}{\rho_{\text{min}}} \right]
\]

\( \mathbf{r} \) is the position operator of the bounded electron
\( \mathbf{v} \) is the velocity of the perturbing electrons
\( f(v) \) is the distribution function of velocities

\( \rho_{\text{max}} \rho_{\text{min}} \) are the limits of the integral over impact parameter in the classical path approximation [Griem et al]

**Weisskopf radius** [Griem et al]:

\[
\rho_{\text{min}} = \frac{n^2 \hbar}{Z m_e v}
\]

\( n \) is the principal quantum number
\( Z \) is the nuclei charge

**En présence du champ magnétique \( \rho_{\text{max}} \) vaut**:

\[
\rho_{\text{max}} = \nu \frac{1}{\sqrt{\left( \Delta \omega^2 + \Delta \omega_p^2 + \Delta \omega_s^2 \right)}} \]

\[
\rho_{\text{max}} = \nu \left( \frac{1}{\omega_L} \right) + \nu \frac{v}{\omega_L}
\]
Δω Is the frequency separation in absence of any perturbation, and Δωp the electron plasma frequency

Δωs Is an estimation of the frequency shift due to linear Stark effect

Δωz Is the frequency separation of two extreme Zeeman components

ω0 Is the central frequency of the line

By using expressions of ρmax and ρmin the collision operator takes the following form:

\[ \Phi \approx -\left(\frac{4\pi}{3}\right)\left(\frac{2m}{\pi k_B T}\right)^{1/2} \frac{n_e}{N_e} \left(\frac{\hbar}{m}\right)^2 \vec{r} \cdot \vec{r} \left(\frac{1}{2} \int \exp(-x) dx\right) \]

where

\[ y \approx \left(\frac{\hbar n^2}{2Z}\right)^2 \frac{\Delta \omega^2 + \Delta \omega_p^2 + \Delta \omega_s^2 + \Delta \omega_z^2}{E_H k_B T} \]

Such that \( E_H = \frac{q_e^2}{2a_0} \) is ionisation energy of hydrogen atom

The matrix elements of \( \Phi \) are proportional:

\[ \langle n\ell m|\vec{r} \cdot \vec{r}|n\ell' m'\rangle = \frac{9}{4} n^2 \left[ n^2 - (\ell^2 - \ell + 1) \right] \delta_\ell\ell' \delta_{mn'} \]

Such that \( m = m_\ell \) because the operator in spin independent

The electron collision operator is then diagonal in the basis \( \left\langle n\ell s m_\ell m_s \right\rangle \)
In presence of the magnetic field:

the non consideration of the trajectory effect is valid only when one is in the domain where the radius of Larmor $\rho_L$ is greater than the length of Debye $\rho_D$ (laboratory plasmas), for the rest of the cases, especially: low densities $N_e < 5 \times 10^{14}$ cm$^{-3}$ and for very strong fields $B > 10^8$ G (astrophysical plasmas) the influence of the curvature of the trajectories is not negligible and then we have to take it into account.

The electron collision operator is diagonal in the basis $(|n\ell sm\ell ms\rangle)$

The diagramm $(N_e,|B|)$ presents a description that allows us to precise the order of magnitude. Also it allows to precise the ranges of the validity of the various hypothesis. For example the non consideration of the trajectory effect is appropriate for certains laboratory plasmas.

Références